LAGUERRE'S POLYNOMIAL

Q. Write Rodrigue's formula for Laguerre's polynomial.

Q. Derive Rodrigue for Laguerre's polynomial.

Q. Show that $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(x^n e^{-x})$

Q. Find value for $L_0(x)$, $L_1(x)$ and $L_2(x)$.

Q. Find value for $L_5(x)$.

RODRIGUE'S FORMULA FOR LAGUERRE POLYNOMIALS:-

The Rodrigue's formula for Laguerre polynomials is,

$$L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(x^n e^{-x}), \text{ n is an integer.}$$

PROOF :- Consider,

$$\frac{d^n}{dx^n}(x^n e^{-x})$$

Differentiating n times with respect to x by using Leibnitz's Theorem; we get

$$\frac{d^n}{dx^n}(x^n e^{-x}) = D^n x^n \cdot D^0 e^{-x} + n D^{n-1} x^n \cdot D' e^{-x} + \frac{n(n-1)}{2!} D^{n-2} x^n \cdot D^2 e^{-x} + \frac{n(n-1)(n-2)}{3!} D^{n-3} x^n \cdot D^3 e^{-x} + \cdots$$

$$= \{n(n-1)(n-2)\cdots 3.2.1.x^0\} e^{-x} + n[n(n-1)(n-2)\ldots 3.2.x](-e^{-x}) + \frac{n(n-1)}{2!}[n(n-1)(n-2)\ldots 4.3.x^2](e^{-x}) + \frac{n(n-1)(n-2)}{3!}[n(n-1)(n-2)\ldots 5.4.x^3](-e^{-x}) + \cdots$$

$$= n! e^{-x} - \frac{n(n-1)!}{(n-1)!} n!\, x e^{-x} + \frac{n(n-1)(n-2)!}{2!(n-2)!}\frac{n!}{2!} x^2 e^{-x} - \frac{n(n-1)(n-2)(n-3)!}{3!\,(n-3)!}\frac{n!}{3!} x^3 e^{-x} + \cdots$$

$$= n! e^{-x} - \frac{n!}{(n-1)!} n!\, x e^{-x} + \frac{n!}{2!\,(n-2)!}\frac{n!}{2!} x^2 e^{-x} - \frac{n!}{3!\,(n-3)!}\frac{n!}{3!} x^3 e^{-x} + \cdots$$

Multiplying both side by $\frac{e^x}{n!}$; we get

$$\frac{e^x}{n!}\frac{d^n}{dx^n}[x^n e^{-x}] = 1 + \frac{(-1)n!}{(n-1)!} x + \frac{(-1)^2 n!}{(2!)^2 (n-2)!} x^2 + \frac{(-1)^3 n!}{(3!)^2 (n-3)!} x^3 + \cdots$$

$$\frac{e^x}{n!}\frac{d^n}{dx^n}[x^n e^{-x}] = \sum_{r=0}^{n} (-1)^r \frac{n!}{(r!)^2 (n-r)!} x^r$$

$$\frac{e^x}{n!}\frac{d^n}{dx^n}[x^n e^{-x}] = L_n(x)$$

or

$$L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}[x^n e^{-x}]$$

which is required ~~repres~~ rodrigue's representation of Laguerre's polynomial.

FINDING LAGUERRE'S ~~RODRIGUE~~ POLYNOMIALS –

Using Rodrigue Laguerre polynomial –

$$L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}[x^n e^{-x}]$$

For $n=0$,

$$L_0(x) = \frac{e^x}{0!}(x^0 e^{-x}) = 1$$

For $n=1$,

$$L_1(x) = \frac{e^x}{1!}\frac{d}{dx}(x e^{-x}) = e^x (e^{-x} - x e^{-x})$$

$$L_1(x) = 1-x$$

For $n=2$,

$$L_2(x) = \frac{e^x}{2!}\frac{d^2}{dx^2}(x^2 e^{-x})$$

$$L_2(x) = \frac{1}{2!}(2-4x+x^2)$$

For $n=3$,

$$L_3(x) = \frac{1}{3!}(6-18x+9x^2-x^3)$$

For $n=4$,

$$L_4(x) = \frac{1}{4!}(24-96x+72x^2-16x^3+x^4)$$